CubeSat Attitude Estimation via AUKF using Magnetometer measurements and MRPs

F. Sanfedino, M. Scardino, Jérémie Chaix and Stéphanie Lizy-Destrez

Abstract In this article the Attitude and Control system of a CubeSat is presented. The attitude estimation design approach used is based on Adaptative Unscented Kalman Filter (AUKF) using three-axis magnetometer measurements. A set of modified Rodrigues Parameters (MRPs) is used to evaluate the attitude. Finally in order to have an complete ADCS system two control laws are introduced (Bdot and Sliding Mode) to best simulate a real CubeSat mission. The first one allows the spacecraft the control during the detumbling phase (phase at high angular rates) and in case of reaction wheels saturation and the second one is used for the nominal control (phase at low angular rates).

1 Introduction

During the last decades there has been a great development of cheap and small satellites, especially in CubeSat projects.
The CubeSat concept created in 1999 by Jordi Puig-Suari of California Polytechnic State University and Bob Twiggs of Stanford University in order to allow students

Francesco SANFEDINO (Student)
Institut Supérieur de l'Aéronautique et de l'Espace, 10, Avenue Edouard-Belin, 31055 Toulouse
e-mail: FRANCESCO.SANFEDINO@SUPAERO.ISAE.FR

Marco SCARDINO (Student)
Institut Supérieur de l'Aéronautique et de l'Espace, 10, Avenue Edouard-Belin, 31055 Toulouse
e-mail: MARCO.SCARDINO@SUPAERO.ISAE.FR

Jérémie CHAIX (Scientist)
Institut Supérieur de l'Aéronautique et de l'Espace, 10, Avenue Edouard-Belin, 31055 Toulouse
e-mail: JEREMIE.CHAIX@ISAE.FR

Stéphanie LIZY-DESTREZ (Associated Professor)
Institut Supérieur de l'Aéronautique et de l'Espace, 10, Avenue Edouard-Belin, 31055 Toulouse
e-mail: STEPHANIE.LIZY-DESTREZ@ISAE.FR

to achieve all the skills which a complex satellite project needs [1].
CubeSats are picosatellites of standardised dimensions (cubes of $10\,cm$ per side with a maximum mass of $1\,kg$). The standard $10x10x10\,cm$ basic CubeSat is often called one unit or 1U CubeSat. CubeSats are scalable along only one axis by 1U increments. As in all satellites also in CubeSats the Attitude and Determination Control System (ADCS) plays a key role in their lives, because it guarantees the right pointing for the communication with the ground station.
In a CubeSat application the ADCS is based above all on three-axis magnetometer employment. This sensor has several advantages such as relative low cost, low required power and continuous availability. In fact the major part of CubeSat orbits are Low Earth Orbit (LEO) and for these ones, during solar eclipse, only the Earth's magnetic field observations are available. Besides magnetometers can also serve as backup attitude estimators [2].
An example of CubeSat which exploits this idea is the JumpSat. The JumpSat is a 3U CubeSat mission proposed by ISAE Supaero in collaboration with TELECOM Bretagne, Massachusetts Institute of Technology (MIT), Centre National dEtude Spatial (CNES) and ONERA.
The goals of this mission are [3]

- Technological verification and Space qualification of a star tracker, which is currently under development by ISAE Supaero for future use in small satellite systems.
- Mapping of the properties of the Earth radiation belt with emphasis of the South Atlantic Anomaly using a directional radiation sensor under development by ONERA.
- Technological verification and Space qualification of the three-axis attitude control system of the Jumpsat space segment.

The ADCS is the system which has the role of satellite attitude control in each phase of its life-cycle. The information taken from some sensors is exploited by actuators in order to produce correction torques. The way of control is based on the specific operational phase.
In the JumpSat mission several phases have been identified.
The most influent ones are two [3]:

- *Rotational Rate Reduction Mode* (Angular rates higher than $5\,^\circ/s$): an operational mode to eliminate the rotational energy of the system after separation from the lunch vehicle or after idle times of the system. It is based on the B-dot control law [4] and utilizes the Magnetometer and Magnetorquer only.
- *Attitude Acquisition Mode* (Angular rates smaller than $5\,^\circ/s$): the main operation mode of the ACS system, based on all available sensors and actuators. It allows pointing of the satellite in any direction in any of the reference frame.

The attention has been focused on the *Rotational Rate Reduction Mode*, which guarantees the satellite mission survival. Thus the model of a real magnetometer has been made by adding a noise and a bias term to the magnetic field. Thanks to this model and a Kalman Filter employment an estimation of the system states (attitude

and angular rate) can be obtained.
For this non-linear application it is necessary to use the Unscented Kalman Filter (UKF) algorithm, based on the Unscented Transformation (UT) [5]. The UT uses a set of sigma points in order to compute the statistics behaviour (propagation of means and covariance) of variables undergone to a nonlinear transformation. Sigma points can be selected according to the symmetric and spherical simplex sigma point [6].
In the next sections the implemented methods and the simulation results will be presented.

2 Methods

2.1 Unscented Kalman Filter

2.1.1 Unscented Transformation

The Unscented Transformation is a method for calculating the statistics of random variable, which undergoes under a non linear transformation. In order to do this, it uses a set of sigma points that guarantees the propagation of means and covariance through the non linear equations. Supposing a random variable $x \in R^n$ has mean $\bar{x}$ and covariance P_x, and x is propagated trough a non linear function, $y = g(x)$. In order to calculate the statistics of y, the sigma points can be selected according to the symmetric and spherical simplex sigma points. For the symmetric sigma points it is necessary to have $2n$ sigma points to represent the mean and covariance, while for the spherical simplex sigma points it is necessary to have $n+2$ points. Generally, the computational cost of Unscented Transformation are proportional to the number of sigma points. For this reason the spherical sigma points approach is chosen. In order to evaluate the spherical silmplex sigma points, the algorithm shows [6] is used.

2.1.2 State estimation

The Unscented Transformation and the sigma points permit the estimation of a non linear dynamic system state vector.

An example of this system in discrete time is:

$$\begin{cases} \mathbf{x}_k = \mathbf{F}(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}) \\ \mathbf{y}_k = \mathbf{H}(\mathbf{x}_k, \mathbf{v}_k) \end{cases} \tag{1}$$

where $\mathbf{x}_k$ represents the states of the system, $\mathbf{y}_k$ is the measurement of the system, $\mathbf{w}_k$ and $\mathbf{v}_k$ are noise, respectively, of the system and of the measurement. The non-linear dynamic equation system considered in this work is:

$$\begin{cases} \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \\ \mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k \end{cases} \tag{2}$$

For this particular non-linear system the Unscented Kalman Filter is called *Additive Unscented Kalman Filter*. The *Additive Unscented Kalman Filter*, respect to classical Unscented Kalman Filter used for the non-linear system 1, provides a greater estimation error, but it is more difficult to tune. The formulation for *Additive Unscented Kalman Filter* is given as follows.

Firstly, the filter is initialized as:

$$\hat{\mathbf{x}}_0 = E[\mathbf{x}_o] = [\mathbf{x}_o]^T \tag{3}$$

$$P_0 = E[(\mathbf{x}_o - \hat{\mathbf{x}}_0)(\mathbf{x_o} - \hat{\mathbf{x}}_0)^T] = diag(P_o) \tag{4}$$

Then the predicted state mean and covariance are computed using Unscented Transformation:

$$\chi_{i,k|k-1} = \mathbf{f}(\chi_{i,k-1}) \quad i = 0, ..., n+1 \tag{5}$$

$$\hat{\mathbf{x}}_k^- = \sum_{i=0}^{n+1} w_i^m \chi_{i,k|k-1} \tag{6}$$

$$P_{x_{k+1}}^- = \sum_{i=0}^{n+1} w_i^c (\chi_{i,k|k-1} - \hat{\mathbf{x}}_k^-)(\chi_{i,k|k-1} - \hat{\mathbf{x}}_k^-)^T + Q_k \tag{7}$$

where χ represents the matrix of sphercial simplex sigma points. The mean and covariance observations are found by:

$$\mathbf{Y}_{i,k} = h(\chi_{i,k|k-1}) \quad i = 0, ..., n+1 \tag{8}$$

$$\hat{\mathbf{y}}_k^- = \sum_{i=0}^{n+1} w_i^m \mathbf{Y}_{i,k} \tag{9}$$

$$P_{y_k} = \sum_{i=0}^{n+1} w_i^c (\mathbf{Y}_{i|k} - \hat{\mathbf{y}}_k^-)(\mathbf{Y}_{i|k} - \hat{\mathbf{y}}_k^-)^T + R_k \tag{10}$$

where Q_k and R_k are the covariance matrix noise, respectively, of the state and measurement. The cross correlation covariance is calculated using:

$$P_{x_k y_k} = \sum_{i=0}^{n+1} w_i^c (\chi_{i,k} - \hat{\mathbf{x}}_k^-)(\mathbf{Y}_{i,k} - \hat{\mathbf{y}}_k^-)^T \tag{11}$$

Finally, the correction stage is defined as follows:

$$K_k = P_{x_k y_k} P_{y_k}^{-1} \tag{12}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + K_k(\mathbf{y}_k - \hat{\mathbf{y}}_k) \tag{13}$$

$$P_{x_k} = P_{x_k}^- - K_k P_{y_k} K_k^T \tag{14}$$

2.1.3 The adaptive tuning of the Q Matrix

The estimation error of UKF depends on the initial choice of the covariance matrix noise Q and of the measurements covariance matrix. For this reason it is necessary to present the Adaptive Unscented Kalman Filter (AUKF), that is a method based on UKF, but the value of Q changes at each step time, in order to reduce the estimation error. An each time step the observation of Q can be written as [7]:

$$Q^* = \Delta \mathbf{x_{k+1}} \Delta \mathbf{x_{k+1}^T} + P_k^- - P_k + Q_k \tag{15}$$

where Q_k is the current covariance matrix noise and $\Delta \mathbf{x_{k+1}}$ is the difference between the estimated and the predicted state.

$$\Delta \mathbf{x_{k+1}} = \hat{\mathbf{x}}_\mathbf{k} - \hat{\mathbf{x}}_k^- \tag{16}$$

So the estimation for the covariance matrix noise is

$$Q_{k+1} = Q_k + \frac{1}{\gamma}[Q^* - Q_k] \tag{17}$$

where γ represents the window size that sets the level of expected change in the noise covariance.

2.2 Attitude dynamics and sensor models

This section provides a brief review of spacecraft attitude dynamics. The attitude parameters here introduced are the Modified Rorigues Parameters. A quaternion system is generally applied for spacecraft pointing and regulation thanks to the absence of singularities in its kinematic equations. However, the use of quaternions requires an extra parameter which leads to a non-minimal parametrization. The Rodrigues parameters provide a minimal (i.e., three dimensional) parametrization. However, a singularity exists for 180° rotations, which hinders this parametrization for extremely large angle rotations. The compromise between the two models it is the modified Rodrigues parameters application, whose singularity at 360° can be solved by a method explained in the section 2.2.1. Moreover they answer to the minimal parametrization need.
As in [8] this parametrization is derived by employing a stereographic projection of the quaternions. The quaternion representation is given by:

$$\mathbf{q} \equiv \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} \tag{18}$$

with

$$\mathbf{q}_{13} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \hat{\mathbf{n}} \sin\left(\frac{\theta}{2}\right) \tag{19}$$

$$q_4 = \cos\left(\frac{\theta}{2}\right) \tag{20}$$

where $\hat{\mathbf{n}}$ is the unit vector corresponding to the axis of rotation and θ is the angle of rotation.
The Modified Rodrigues Parameters are defined by Equation (21):

$$\mathbf{p} = \frac{\mathbf{q}_{13}}{1+q_4} = \hat{\mathbf{n}} \tan\left(\frac{\theta}{4}\right) \tag{21}$$

where $\mathbf{p}$ is a 3×1 vector. The kinematic equations of motion are derived by using the spacecraft's angular velocity $\mathbf{w}$, given by:

$$\dot{\mathbf{p}} = \frac{1}{2}\left\{\frac{1}{2}\left(1-\mathbf{p}^T\mathbf{p}\right) I_{3\times 3} + [\mathbf{p}\times] + \mathbf{p}\mathbf{p}^T\right\}\mathbf{w} \tag{22}$$

The non-linear three-axis rotational dynamics of the rigid spacecraft with momentum wheel may be expressed as:

$$\dot{\mathbf{w}} = J^{-1}\left\{\mathbf{T}_c - \dot{\mathbf{H}}_i - [\mathbf{w}\times]\mathbf{H}_i - [\mathbf{w}\times] J\mathbf{w} + \Delta\mathbf{T}\right\} \tag{23}$$

where, J is the moment of the inertia matrix, $\mathbf{T}_c$ is the magnetorquer control torque, $\mathbf{H}_i$ is the angular momentum vector and , $\dot{\mathbf{H}}_i$ is the wheel control torque and $\Delta\mathbf{T}$ is the disturbance torque.
For convenience, defining the state vector $\mathbf{x} \in \mathbb{R}^{7\times 1}$ in the attitude estimator as $\mathbf{x} = \left[\mathbf{p}^T \ \mathbf{w}^T \ t\right]^T$, where the time t is added as estimation variable for a simpler implementation.
The non-linear dynamics equation for propagating $\mathbf{x}$ is rewritten as

$$\dot{\mathbf{x}} = F\left(\mathbf{x}, \mathbf{w}_T\right) = \mathbf{f}\left(\mathbf{x}\right) + \mathbf{w}_T \tag{24}$$

where

$$\mathbf{f}\left(\mathbf{x}\right) = \begin{bmatrix} \frac{1}{2}\left\{\frac{1}{2}\left(1-\mathbf{p}^T\mathbf{p}\right) I_{3\times 3} + [\mathbf{p}\times] + \mathbf{p}\mathbf{p}^T\right\}\mathbf{w} \\ J^{-1}\left\{\mathbf{T}_c - \dot{\mathbf{H}}_i - [\mathbf{w}\times]\mathbf{H}_i - [\mathbf{w}\times] J\mathbf{w} + \Delta\mathbf{T}\right\} \\ 1 \end{bmatrix} \tag{25}$$

and the process noise $\mathbf{w}_T$ is zero-mean white noise described by the process noise matrix Q. The attitude measurement model for a single sensor is given by:

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{B_{body}}_k \\ t_k \end{bmatrix} + \mathbf{v}_k = \begin{bmatrix} A\left(\mathbf{p}_k\right) \\ 1 \end{bmatrix} \begin{bmatrix} \mathbf{B}_k \\ t_{OBC} \end{bmatrix} + \mathbf{v}_k \tag{26}$$

where $\mathbf{y}_k$ is the k^{th} measurement vector, t_{OBC} is the time provided by the on-board computer, and $\mathbf{v}_k$ is measurement zeros-mean white noise. So the measurements of the Earth magnetic field in the orbital frame are translated into the spacecraft body frame by using the matrix $A(\mathbf{p})$

$$A(\mathbf{p}) = I_{3\times 3} - \frac{4\left(1-\mathbf{p}^T\mathbf{p}\right)}{\left(1+\mathbf{p}^T\mathbf{p}\right)^2}[\mathbf{p}\times] + \frac{8}{\left(1+\mathbf{p}^T\mathbf{p}\right)^2}[\mathbf{p}\times]^2 \tag{27}$$

2.2.1 How to avoid the singularity of Modified Rodrigues Parameters

In order to avoid the singularity of Modified Rodrigues Parameters, it is possible to use the similar parameters, which are called *shadow* Modified Rodrigues Parameters [9]:

$$p_i^S = -\frac{q_i}{1-q_4} = -\frac{p_i}{\mathbf{p}^T\mathbf{p}} \tag{28}$$

The shadow parameters $\mathbf{p}^S$ have some interesting properties. They have a singularity at the zero rotation and they go to zero at $\pm 360^\circ$ of principal rotation. This is the exact opposite of $\mathbf{p}$. For this reason they can be used when the Modified Rodrigues Parameters go to singular. So with these shadow points it is possible describe any rotation of satellite without singularity, but one discontinuity is present at the switching point. In terms of $\mathbf{p}$ the cosine matrix and the kinematic equation are exactly the same as 22 and 27.

In order to switch between these parameters it is possible to use the following relationship:

$$|\mathbf{p}||\mathbf{p}^s| = 1 \tag{29}$$

When using $\mathbf{p}$ to represent the attitude, there is switch from $\mathbf{p}$ to $\mathbf{p}^s$ if $|\mathbf{p}| > 1$ and thus:

$$\mathbf{p}^s = -\frac{\mathbf{p}}{|\mathbf{p}|^2} \tag{30}$$

While using $\mathbf{p}^s$ to represent the attitude, there is switch from $\mathbf{p}^s$ to $\mathbf{p}$ if $|\mathbf{p}^s| > 1$, and thus:

$$\mathbf{p} = -\frac{\mathbf{p}^s}{|\mathbf{p}^s|^2} \tag{31}$$

So with this definition the magnitude of $\mathbf{p}$ and $\mathbf{p}^s$ will never exceed 1, which results in avoiding the singularity.

2.3 Control law

This section will present the two control laws implemented in this project: the B-Dot for the *Rate-Reduction Mode* and the Sliding Mode for the *Attitude-Acquisition Mode*.

2.3.1 B-Dot Control Law

This section describes the B-Dot control law for the *Rate-Reduction Mode*. The principle on which the B-Dot is based [4] is the minimization of the derivative of the Earth's magnetic field vector **B** measured by a magnetometer. The rate of its change depends on the spacecraft rotation rate. Thus the minimization of this derivative determines a decrease of the satellite angular rate that corresponds to a reduction of the rotational kinetic energy. This is defined as:

$$\dot{\mathbf{E}}_{\mathbf{rot}} = \frac{d}{dt}\left(\frac{1}{2}\mathbf{w_{body}}^T \cdot \mathbf{I_{sat}} \cdot \mathbf{w_{body}}\right) \tag{32}$$

This means that the scalar product of the angular rate of satellite body and the control torque must be smaller than zero:

$$\mathbf{w_{body}}^T \cdot \mathbf{T_c} < 0 \tag{33}$$

The control torque $\mathbf{T_c}$ is the result of the interaction of the Earth magnetic field vector **B** and the magnetorquers magnetic momentum $\mathbf{M_{torquer}}$:

$$\mathbf{T_c} = \mathbf{M_{torquer}} \times \mathbf{B_{Earth}} \tag{34}$$

Thus:

$$\mathbf{w_{body}}^T \cdot (\mathbf{M_{torquer}} \times \mathbf{B_{Earth}}) < 0 \tag{35}$$

After rearranging, Equation (35) becomes:

$$\mathbf{M_{torquer}} \cdot \left(\mathbf{w_{body}}^T \times \mathbf{B_{Earth}}\right) < 0 \tag{36}$$

From this inequality it can be deduced that the unique negative parameter has to be $\mathbf{M_{torquer}}$. Thus a negative control scalar gain C_{b-dot} is introduced in order to minimise the rotation kinetic energy.

The commanded control torque becomes:

$$\mathbf{M_{torquer}} = -C_{b-dot} \cdot (\mathbf{w_{body}} \times \mathbf{B_{Earth}}) \tag{37}$$

In Equation 37 the cross product between the angular body rate $\mathbf{w_{body}}$ and the Earth's magnetic field vector is equal to the time derivative of the Earth's magnetic field vector $\dot{\mathbf{B}}_{\mathbf{Earth}}$:

$$\dot{\mathbf{B}}_{\mathbf{Earth}} = \mathbf{w_{body}} \times \mathbf{B_{Earth}} \tag{38}$$

The control law finally becomes:

$$\mathbf{M_{torquer}} = -C_{b-dot} \cdot \dot{\mathbf{B}}_{\mathbf{Earth}} \tag{39}$$

2.3.2 Synthesis of Sliding mode control

Considering the following non-linear system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{U} \tag{40}$$

where $\mathbf{x}$ is the state vector of the system and $\mathbf{U}$ represents the command that stabilizes $\mathbf{x}$. For this system the goal is to find the value of $\mathbf{U}$. In order to do this, one approach is to choose a surface in the state space, called sliding surface $\mathbf{S}(\mathbf{x})$, where the command objective is:

- If $\mathbf{S}(\mathbf{x}) = 0$, the value of state vector has to be zero ($\mathbf{x} = 0$)
- To bring the state vector from an arbitrary position to the sliding surface.

Besides, the command of sliding mode can be characterized by the principle of attractiveness ($\mathbf{S}(\mathbf{x})\dot{\mathbf{S}}(\mathbf{x}) < 0$) and of invariance ($\dot{\mathbf{S}}(\mathbf{x})) = 0$ for $\mathbf{S}(\mathbf{x})) = 0$). With these principles it is possible to compute the value of $\mathbf{U}$ [10]:

$$\mathbf{U} = -\left(\left(\frac{\delta\mathbf{S}}{\delta\mathbf{x}}\right)^T \mathbf{g}(\mathbf{x})\right)^{-1}\left(\left(\frac{\delta\mathbf{S}}{\delta\mathbf{x}}\right)^T \mathbf{f}(\mathbf{x})\right) - Ksat(\mathbf{S}(\mathbf{x},\varepsilon)) \tag{41}$$

where K is a diagonal matrix that permits $\mathbf{U}$ to assure the condition of attractiveness and $sat(\mathbf{S}(\mathbf{x}))$ is the saturation function, that generally is equal to:

$$sat(\mathbf{S}(\mathbf{x},\varepsilon)) = \begin{cases} -1 & if \quad \mathbf{S}(\mathbf{x}) < -\varepsilon \\ \frac{\mathbf{S}(\mathbf{x})}{\varepsilon} & if \quad |\mathbf{S}(\mathbf{x})| < \varepsilon \\ 1 & if \quad \mathbf{S}(\mathbf{x}) > \varepsilon \end{cases} \tag{42}$$

Sliding Surface using Modified Rodrigues and reaction wheels

For the *operational mode*, only the reaction wheels represent the way to stabilize the JumpSat. For this reason the control torque of magnetotorques is equal to zero and the Equation (23) of dynamics becomes:

$$J\dot{\mathbf{w}} + [\mathbf{w}\times]J\mathbf{w} = -[\mathbf{w}\times]\mathbf{H}_i - \dot{\mathbf{H}}_i + \Delta\mathbf{T} \tag{43}$$

Finally the linear model for spacecraft motion is:

$$\dot{\mathbf{p}} = \mathbf{F}(\mathbf{p})\mathbf{w} \tag{44}$$

$$\dot{\mathbf{w}} = \mathbf{f}(\mathbf{x}) - J^{-1}\mathbf{U} - J^{-1}\Delta\mathbf{T} \tag{45}$$

where,

$$\mathbf{F}(\mathbf{p}) = \frac{1}{2}\left\{\frac{1}{2}\left(1-\mathbf{p}^T\mathbf{p}\right)I_{3\times 3} + [\mathbf{p}\times] + \mathbf{p}\mathbf{p}^T\right\} \tag{46}$$

$$\mathbf{f}(\mathbf{p}) = -J^{-1}[\mathbf{w}\times]J\mathbf{w} \tag{47}$$

$$\mathbf{U} = -[\mathbf{w}\times]\mathbf{H}_i - \dot{\mathbf{H}}_\mathbf{i} \tag{48}$$

For this dynamics the sliding surface is[11]:

$$\mathbf{S}(\mathbf{p}) = \mathbf{w} - \mathbf{m}(\mathbf{p}) \tag{49}$$

The value of $\mathbf{m}(\mathbf{p})$ is calculated from a desired vector of Modified Rodrigues Parameters and the kinematics equation:

$$\mathbf{m}(\mathbf{p}) = \mathbf{F}^{-1}(\mathbf{p})\mathbf{d}(\mathbf{p}) \tag{50}$$

where,

$$\begin{aligned}\mathbf{F}^{-1}(\mathbf{p}) =& 4(1+\mathbf{p}^T\mathbf{p})^{-2}\\ &\left\{(1+\mathbf{p}^T\mathbf{p})I_{3\times3} - 2[\mathbf{p}\times] + 2\mathbf{p}\mathbf{p}^T\right\}\end{aligned}$$

and

$$\mathbf{d}(\mathbf{p}) = \Lambda(\mathbf{p} - \mathbf{p}_d) \tag{51}$$

where $\mathbf{P}_d$ is the desired reference of Modified Rodrigues Parameters and Λ is a diagonal matrix with negative elements. So with these elements the value of command is:

$$\begin{aligned}\mathbf{U} =& -J\left\{\mathbf{f}(\mathbf{w}) - \frac{\delta\mathbf{m}}{\delta\mathbf{p}}[\mathbf{F}(\mathbf{p})\mathbf{m}(\mathbf{p}) + \mathbf{F}(\mathbf{p})\mathbf{S}(\mathbf{p})]\right\}\\ &- JKsat(\mathbf{S}(\mathbf{p}),\varepsilon)\end{aligned}$$

Supposing that the matrix Λ is given by a scalar λ times the identity matrix, the quantity $\mathbf{m}(\mathbf{p})$ becomes:

$$\begin{aligned}\mathbf{m}(\mathbf{p}) =& 4\lambda\left(1+\mathbf{p}^T\mathbf{p}\right)^{-1} - 4\lambda\left(1+\mathbf{p}^T\mathbf{p}\right)^{-2}\\ &\left\{\left(1-\mathbf{p}^T\mathbf{p}\right)I_{3\times3} - 2[\mathbf{p}\times] + 2\mathbf{p}\mathbf{p}^T\right\}\mathbf{p}_d\end{aligned}$$

ans its derivative respect $\mathbf{p}$ is equal to:

$$\begin{aligned}\frac{\delta\mathbf{m}(\mathbf{p})}{\delta\mathbf{p}} =& 4\lambda\left(1+\mathbf{p}^T\mathbf{p}\right)^{-1}\left\{I_{3\times3} - 2\left(1+\mathbf{p}^T\mathbf{p}\right)^{-1}\mathbf{p}\mathbf{p}^T\right\}\\ &- 8\lambda\left(1+\mathbf{p}^T\mathbf{p}\right)^{-2}\left\{\mathbf{p}\mathbf{p}_d^T - \mathbf{p}_d\mathbf{p}^T + [\mathbf{p}_d\times] + \left(\mathbf{p}_d^T\mathbf{p}\right)I_{3\times3}\right\}\\ &+ 16\lambda\left(1+\mathbf{p}^T\mathbf{p}\right)^{-3}\left\{\left(1-\mathbf{p}^T\mathbf{p}\right)I_{3\times3} - 2[\mathbf{p}\times] + 2\mathbf{p}\mathbf{p}^T\right\}\mathbf{p}_d\mathbf{p}^T\end{aligned}$$

Finally in order to eliminate the effects of external disturbances, it is necessary to add to the quantity $\mathbf{U}$ another torque $\mathbf{U}_{dist}$, so the command to stabilise the system becomes:

$$\mathbf{U}_{tot} = \mathbf{U} + \mathbf{U}_{dist} \tag{52}$$

where

$$\mathbf{U}_{dist} = -\Delta\hat{\mathbf{T}} \tag{53}$$

$\Delta\hat{\mathbf{T}}$ is generally computed by an observer, but in our case it is a value computed by the *Unscented Kalman Filter*.

3 Results and Simulation

In this section two implemented models will be presented. Firstly the efficiency of the UKF estimation without the introduction of the control law will be analysed in order to give confidence to the measurements of the three-axis magnetometers. Then the results of the controlled system will be presented. For the simulations a model of the Earth magnetic field has been introduced according to the specification of IGRF11 available until 2015 [12]. The magnetic field vector has been computed at each $0.1\,s$ interval. In the reality only the measurements of the sensor will be used in order to evaluate the spacecraft attitude. The model of the magnetometer consists in a sensor with a bias and a white Gaussian noise to best simulate the errors accumulated by a real instrument. According to the verification of UKF algorithm all the control part has been removed. For the JumpSat mission a system of reaction wheels and magnetorquers is used to actuate the control. The elimination of the control system has as consequence the suppression of the wheel control torque $\dot{\mathbf{H}}_i$ and of the angular momentum vector $\mathbf{H}_i$, which have an important role in the dynamics Equation (23), but also of $\mathbf{T_c}$.

The complete system scheme is presented in Figure 1, where also the control law block is considered.

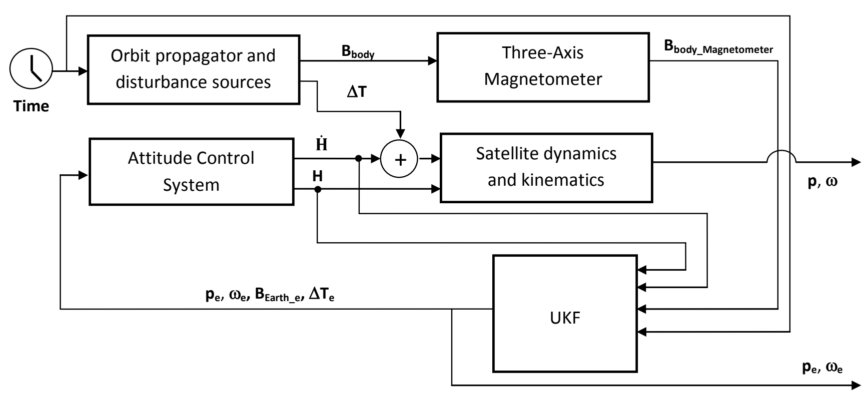

Fig. 1 Simulation scheme

For the test of estimation the blocks architecture except for the UKF are heritage of Christoph Pierls work [13], but all the equations has been translated into the

Parameter	*Value*
J	$diag[0.045, 0.045, 0.005]\, kg\, m^2$
$\mathbf{B_{bias}}$	$[25, -25, 25]^T\, nT$
$\mathbf{B_{res\,dip}}$	$[5\cdot 10^{-8}, 5\cdot 10^{-8}, 5\cdot 10^{-6}]\, Am^2$
$\mathbf{B_{pds}}$	$[2\cdot 10^{-9}; 2\cdot 10^{-9}; 2\cdot 10^{-9}]\, T$
W_0	0.5
P_0	$diag[0.087; 0.087; 0.87; 1; 1; 1; 0.1]\cdot 10^{-1}$
Q_0	$diag[1.563; 1.43; 1.984; 0; 0; 0; 0.3]\cdot 10^{-14}$
R_0	$diag[25; 25; 25; 100]\cdot 10^{-15}$
$\mathbf{p_0}$	$[0; 0.414; 0]$
$\mathbf{w_0}$	$[0; 0; 0]\, rad/s$
γ	50
λ	-0.015
ε	0.01
k	0.0015
C_{b-dot}	-20000

Table 1 Parameters used in the estimation model. $\mathbf{B_{bias}}$ is the bias on the three-axis magnetometer, $\mathbf{B_{res\,dip}}$ is the residual dipole momentum of the satellite. $\mathbf{B_{pds}}$ is the deviation standard of the magnetic noise.

modified Rodrigues Parameters $\mathbf{p}$ from the quaternion system and the control part has been adapted to the current case.
The *Orbit propagator and disturbance sources* block takes the time to calculate the evolution of the spacecraft trajectory in its orbit. Thanks to the orbit parameters evaluation it is possible to obtain the Earth magnetic field in the body frame $\mathbf{B_{body}}$ (here calculated by the IGRF11 model for a better precision) and all the considerable disturbing torques $\Delta\mathbf{T}$, i.e. the magnetic and the gravity gradient ones. The block *Satellite dynamics and kinematics* comprehends the Equations (22) and (23). The *Attitude Control System* takes into account the control law and the reaction wheel model in order to provide the control torque $\dot{\mathbf{H}}_i$ and the angular momentum $\mathbf{H}_i$.

For all the simulations two important hypothesis have been established:

- The Earth magnetic field model is the same in the block *Orbit propagator and disturbance sources* and in the filter model. In the reality a simpler algorithm than the IGRF11 is used in the embedded code for computational reasons. Thus it introduces other uncertainties not considered here.
- The model of the orbit is not estimated but taken from the environment modelled in the block *Orbit propagator and disturbance sources*. Thus a development of this work would be the implementation of an algorithm which predicts the satellite position in its orbit.

A set of parameters has been chosen to best produce the estimation. They are listed in Table 1.

In the following graphs different simulation of the implemented methods are presented. Particularly the variation of Euler's angles with the time will present . These angles represent the rotation needed to bring the body frame to the orbital frame:

- The angle ψ is the rotation around the axis Z_{body}, which brings the body frame to an intermediate frame identified with $X1_{body}$, $Y1_{body}$ and $Z1_{body} = Z_{body}$. This rotation is considered positive if it is an anti-clockwise rotation.
- The angle θ is the rotation around $Y1_{body}$, which brings the $F1_{body}$ to another intermediate frame, $X2_{body}$, $Y2_{body} = Y1_{body}$ and $Z2_{body}$. This rotation is considered positive if it is an anti-clockwise rotation.
- The angle ϕ is the rotation around $X2_{body} = X_{orbital}$, which aligns the two frames. It is positive if it is an anti-clockwise rotation.

An algorithm permit us to compute these angles from the values of the Modified Rodrigues Parameters. The first step of this algorithm is the evaluation of the quaternions:

$$q_4 = \frac{1-\mathbf{p}^T\mathbf{p}}{1+\mathbf{p}^T\mathbf{p}} \quad q_i = \frac{2p_i}{1+\mathbf{p}^T\mathbf{p}} \quad for \quad i = 1,2,3 \tag{54}$$

With the values of the quaternions the Euler's angles are equals to:

$$\psi = atan\left(\frac{2(q_4q_3 + q_1 + q_2)}{1 - 2(q_2^2+q_3^2)}\right) \tag{55}$$

$$\theta = asin\left(2(q_4q_2 - q_3q_1)\right) \tag{56}$$

$$\phi = atan\left(\frac{2(q_4q_1 + q_2q_3)}{1 - 2(q_1^2+q_2^2)}\right) \tag{57}$$

The limitations of this algorithm implemented on Matlab is that the values of ϕ and ψ are comprised between -180° and 180° and for θ the values are comprised between -90° and 90°.

3.1 Performances of Modified Rodrigues Parameters

In this section a simulations will be presented in order to evaluate the filter efficiency.

Tacking as initial condition,

$$(\psi,\theta,\phi) = (140^\circ, 90^\circ, 160^\circ) \tag{58}$$

$$(r,q,p) = (1.3^\circ/s, 1^\circ/s, 0.85^\circ/s) \tag{59}$$

the estimation of UKF with Modified Rodrigues Parameters is presented in Figure 2, Figure 3 and Figure 4 for a simulation time of $1000s$ and using the IGRF11 model for the Earth's magnetic field.

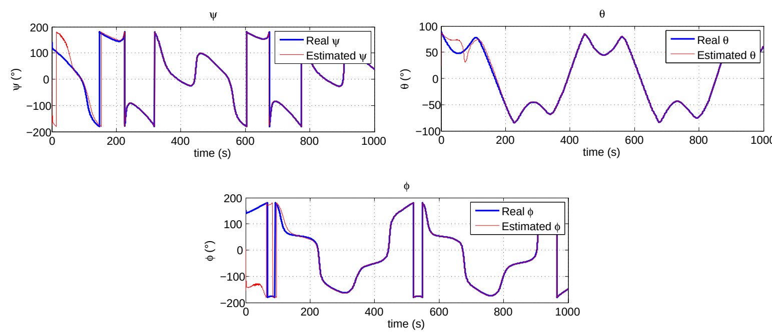

Fig. 2 Comparison between the estimate Euler angles and real value.

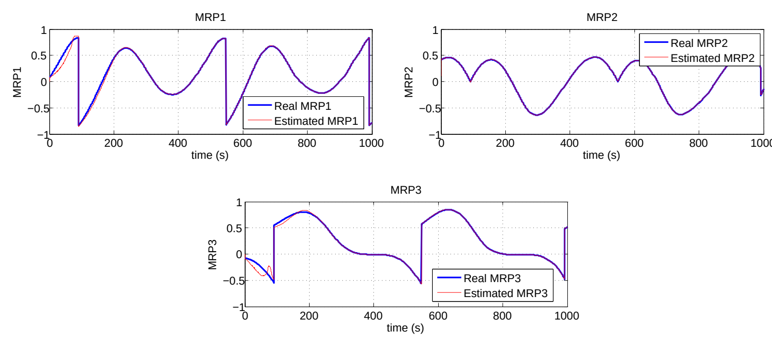

Fig. 3 Comparison between the estimate Modified Rodrigues Parameters value and real value.

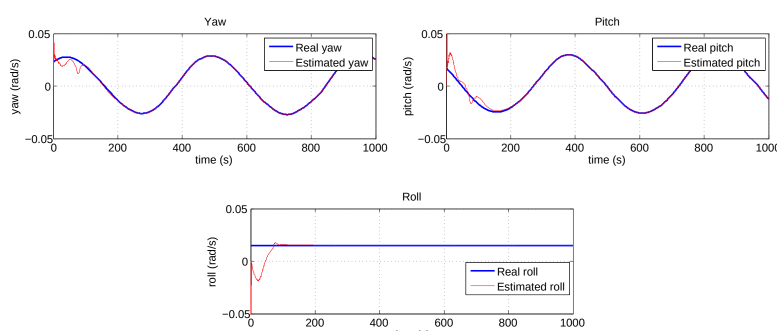

Fig. 4 Comparison between the estimate Angular rates value and real value.

3.2 Interest of the adaptive method

This section shows as the adaptive method for the noise covariance matrix guarantees better results in terms of errors between the real system and the estimated one. Tacking as initial condition,

$$(\psi,\theta,\phi) = (140^\circ, 90^\circ, 160^\circ) \tag{60}$$

$$(r,q,p) = (1.3^\circ/s, 1^\circ/s, 0.85^\circ/s) \tag{61}$$

a comparison between normal UKF and its adaptive version is shown in Figure 5.

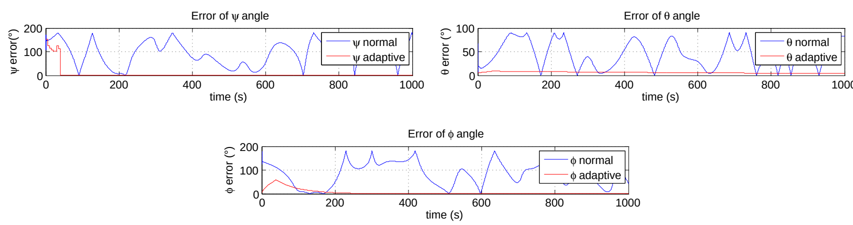

Fig. 5 Error between classical and adaptive UKF

3.3 Attitude Control

This section provides the complete control attitude simulation. Both B-Dot and Sliding Mode are applied: the first one in order to reduce the angular rates during the detumbling phase and in case of the reaction wheels saturation (*Rate-Reduction-Mode*). The second one is then used to reach the need attitude (*Attitude-Acquisition-Mode*), which corresponds to the condition 0° for all three Euler's angles.

For the simulation the magnetorquers provide a nominal magnetic momentum of $0.2\,Am^2$ and the reaction wheel have a saturation torque of $0.635\,mN\,m$. The switch from a control law to the other one consists in the condition suggested in [13]: intervention of B-Dot if all the angular rates are bigger than $5^\circ/s$. In case of switch a little retard $(0.05\,s)$ is introduced to permit the system a fluent passage from a mode to the other one without discontinuities.

Tacking as initial conditions:

$$(\psi,\theta,\phi) = (-100^\circ, 30^\circ, -100^\circ) \tag{62}$$

$$(r,q,p) = (8.6^\circ/s, -8.6^\circ/s, 14.3^\circ/s) \tag{63}$$

In Figure 6 the Euler's angles evolution is presented for a simulation time of 9500 s.

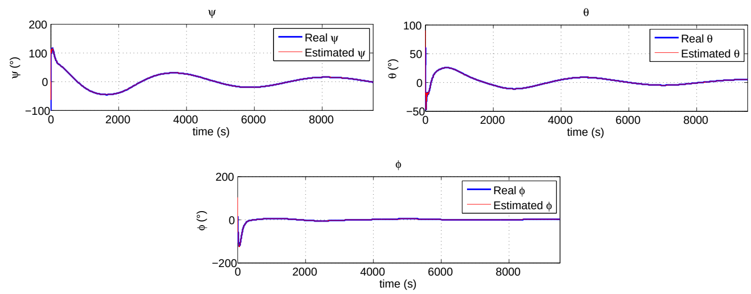

Fig. 6 Euler angles

3.4 Monte Carlo simulation

Monte Carlo simulations are computational algorithms that rely on repeated random sampling to obtain numerical results, typically one runs simulations many times over in order to obtain the distribution of an unknown entity.

The verification of our procedure, based on UKF algorithms and Modified Rodrigues Parameters, will be made with Monte Carlo Simulations of the SIMULINK model with random sets of parameters. The goal is:

- to verify or dismiss the current estimation filter for actual implementation to the space segment.
- to obtain an estimation on the performance of the algorithm.

The assumptions for the Monte Carlo methods are:

- Each simulation has a duration of 3500 s;
- The system is considered converged, if the filter provides an estimation of the space segment attitude with an error of less than 5° around all three axes of the local orbital frame in less than 2400 s and maintains this limit for 900 s

The error ε of Monte Carlo approach is defined as $\varepsilon = \frac{1}{\sqrt{N}}$, where N is the total number of simulations. In order to provide a result with a 10 % error, a number of 100 simulations are necessary.

For simulating the truth model of satellite a set of random parameters were chosen, particularly the standard deviation value of the sensor σ_B and the bias value of the sensor B_{bias} as the set of random parameters.

But the initial value of the angular rate and satellite attitude were also chosen as random parameters . Particularly the bounds of these two parameters are:

- The angular rate w_z and w_y are comprised between $-8.6°/s$ and $8.6°/s$, while the w_x between $-14.3°/s$ and $14.3°/s$.
- The attitude value of ψ and ϕ are comprised between $-180°$ and $180°$, while the angle θ is comprised between $-90°$ and $90°$.

3.4.1 Results of Monte Carlo Simulation

The Monte Carlo simulation was run with the highest number of iterations possible with the given limitation on time, 153 times. Among these 153 simulations, 144 lead to success so:

$$P_{convergence} = \frac{144}{153} = 94.12\,\% \tag{64}$$

The average time needed for the system to reach convergence is equal to:

$$t_{average_{conv}} = \frac{\sum_{i=0}^{N_{convergence}} t_{i_{conv}}}{N_{convergence}} = 131.2\,s \tag{65}$$

In the Figure 7 and 8, it is possible to see the variation of the convergence time with different values of ψ_{ini}, ϕ_{ini}, θ_{ini}, the norm of angular rate($\sqrt{w_x^2+w_y^2+w_z^2}$),norm of the sensor bias ($\sqrt{B_{bias_x}^2+B_{bias_y}^2+B_{bias_z}^2}$) and norm of the sensor deviation standard ($\sqrt{B_{psd_x}^2+B_{psd_y}^2+B_{psd_z}^2}$). These simulations prove that the Adaptive Unscented Kalman filter, using Modified Rodrigues Parameters, is stable for various initial states.

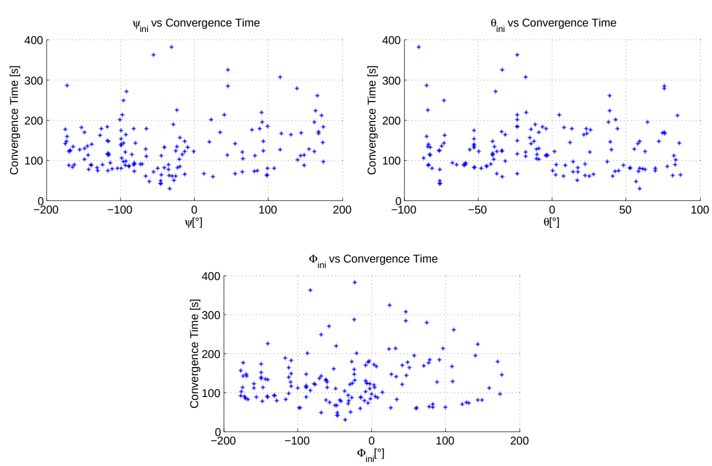

Fig. 7 Resulte of Monte Carlo simulation for Euler's angles.

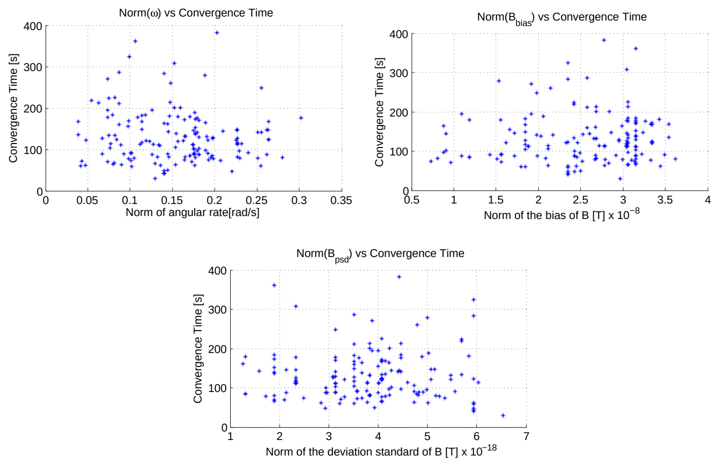

Fig. 8 Results of Monte Carlo simulation for the angular rate norm, the Earth's magnetic bias norm and the Earth's magnetic deviation standard norm.

4 Conclusions

In this paper the UKF advantage in spacecraft attitude estimation were presented and discussed . It was explored the possibility to apply the Modified Rodrigues Parameters and it was verified the efficiency of the estimation algorithm. In order to verify the reliability for future embedded applications a Monte Carlo simulation has been made by changing all the unknown parameters. A large set of initial condition after the orbit injection has been speculated in order to manage to control the satellite in each situation.

At this step of project development some hypothesis have been taken into account to simplify the problem. Future extensions consist as said before in developing also an estimation for the orbit propagator and testing two types of Earth's magnetic field: one more precise (e.g. IGRF11) for the real model and a simplified version for the estimator. This operation results compulsory because of the impossibility to charge the on-board computer with a complex model. Another thing here neglected is the influence of Aerodynamic disturbance torque. Its effect has to be considerate significant especially in detumbling phase, when spacecraft surfaces are largely exposed to the density of the low atmosphere in their rotation. Thus the implementation of a model for this disturbance torque has to be carried out.

References

1. Heidt, H., Puig-Suari, J., Moore, A., Nakasuka, S., and Twiggs, R., "CubeSat: A new generation of picosatellite for education and industry low-cost space experimentation," 2000.

2. Psiaki, M. L., Martel, F., and Pal, P. K., "Three-axis attitude determination via Kalman filtering of magnetometer data," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 3, 1990, pp. 506–514.
3. Lucas, W., Rouanne-Labe, A., Grave, J., Peille, P., and Lizy-Destrez, S., "JUMPSAT: Qualifying three equipments in one Cubesat mission," 2013.
4. Flatley, T., Morgenstern, W., Reth, A., and Bauer, F., "A B-dot acquisition controller for the RADARSAT spacecraft," *NASA Conference Publication*, NASA, 1997, pp. 79–90.
5. Julier, S. J. and Uhlmann, J. K., "Unscented filtering and nonlinear estimation," *Proceedings of the IEEE*, Vol. 92, No. 3, 2004, pp. 401–422.
6. Lozano, J. G. C., Carrillo, L. G., Dzul, A., and Lozano, R., "Spherical simplex sigma-point Kalman filters: a comparison in the inertial navigation of a terrestrial vehicle," *American Control Conference, 2008*, IEEE, 2008, pp. 3536–3541.
7. Soken, H. E. and Sakai, S.-i., "Adaptive Unscented Kalman Filter for Small Satellite Attitude Estimation," 2012.
8. Crassidis, J. L. and Markiey, F., "Attitude Estimation Using Modified Rodrigues Parameters," 1996.
9. Schaub, H. and Junkins, J. L., "Stereographic orientation parameters for attitude dynamics: A generalization of the Rodrigues parameters," *Journal of the Astronautical Sciences*, Vol. 44, No. 1, 1996, pp. 1–19.
10. Bellar, A., Seba, B., MOHAMMED, A., and Sweeting, M., "Tree axis attitude control using sliding mode for LEO microsatellite," .
11. Crassidis, J. L. and Markley, F. L., "Sliding mode control using modified Rodrigues parameters," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 6, 1996, pp. 1381–1383.
12. Finlay, C., Maus, S., Beggan, C., Bondar, T., Chambodut, A., Chernova, T., Chulliat, A., Golovkov, V., Hamilton, B., Hamoudi, M., et al., "International geomagnetic reference field: the eleventh generation," *Geophysical Journal International*, Vol. 183, No. 3, 2010, pp. 1216–1230.
13. Pierl, C., "Preliminary Design of the Attitude Control System and Operational Mode Transition of the JumpSat project," *Final report*, 2013.